电路基础
(Fundamentals of Electric Circuits, INF0120002.07)

2019年03月05日

唐长文 教授

zwtang@fudan.edu.cn

http://rfic.fudan.edu.cn/Courses.htm

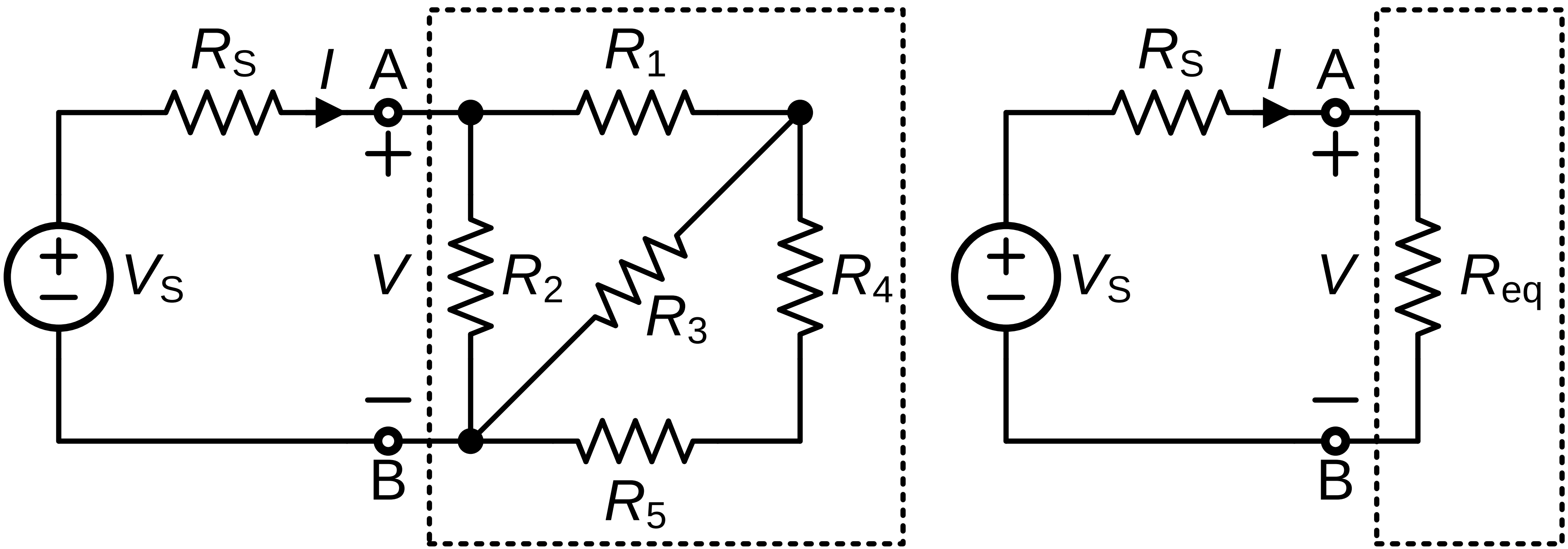
复旦大学/微电子学院/射频集成电路设计研究小组

第二章 电阻电路的等效

- 等效电路
- 串联和并联
- 星形和三角形联结的等效
- 电源和电阻的串联与并联
- 端口等效电阻

等效电路

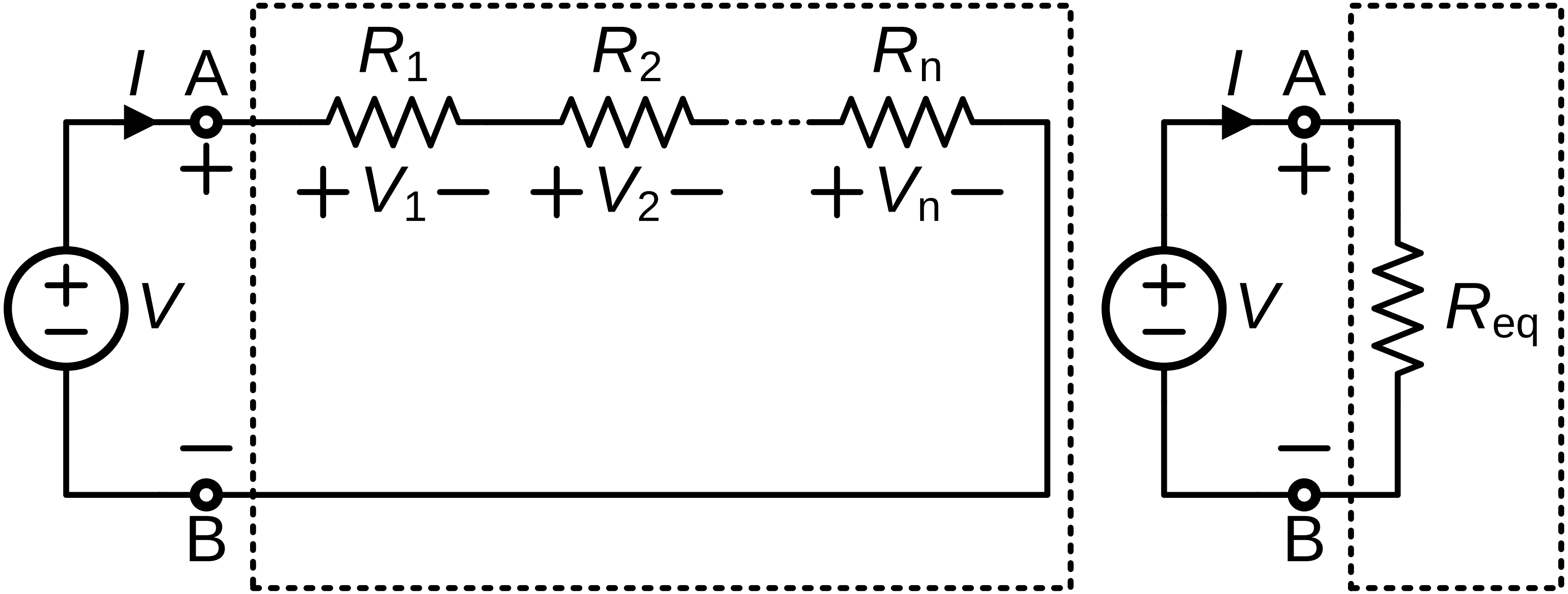
等效电路与被等效电路的端口电压和电流特性保持一致。等效只是“对外等效”！未被替换部分的电压和电流均保持不变。



串联

串联的每个电阻，电流相同，电压与电阻值成正比。

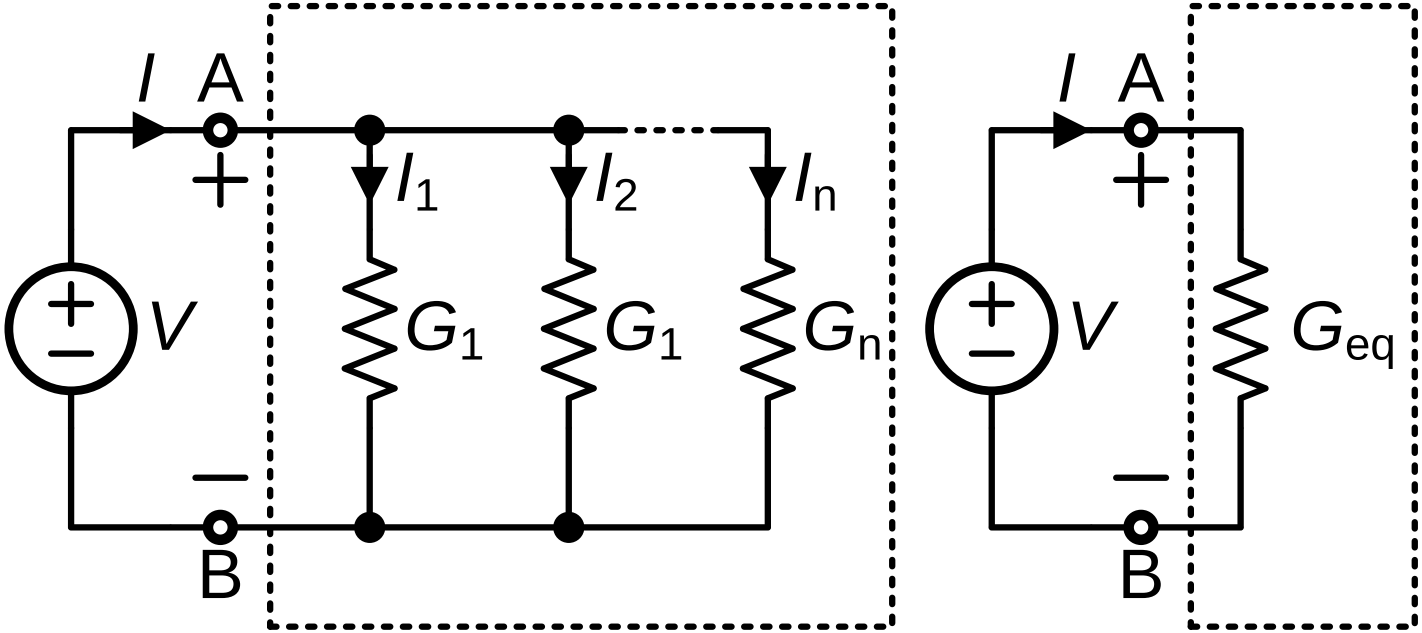
$$R_{eq} = \sum_{k=1}^{n} R_k \qquad V_k = V\frac{R_k}{R_{eq}}$$



并联

并联的每个电阻，电压相同，电流与电导值成正比。

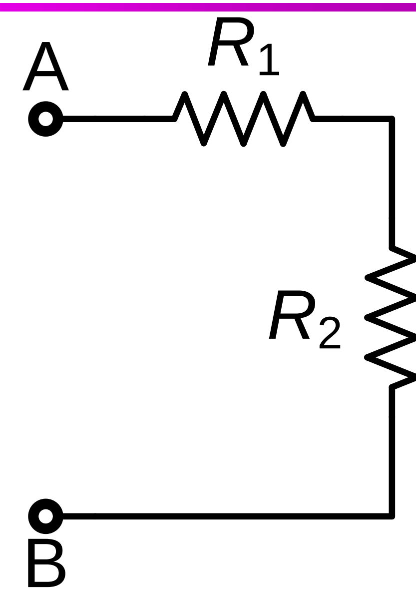
$$G_{eq}=\sum_{k=1}^{n}G_k \qquad I_k=I\frac{G_k}{G_{eq}}$$



两个电阻的串联和并联

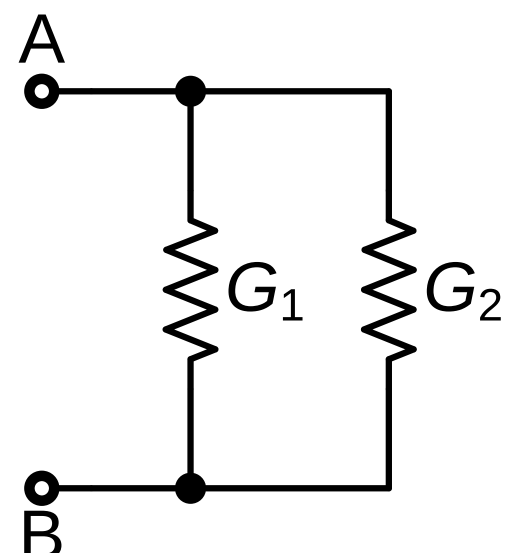
- 串联

$$R_{eq} = R_1 + R_2 \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$



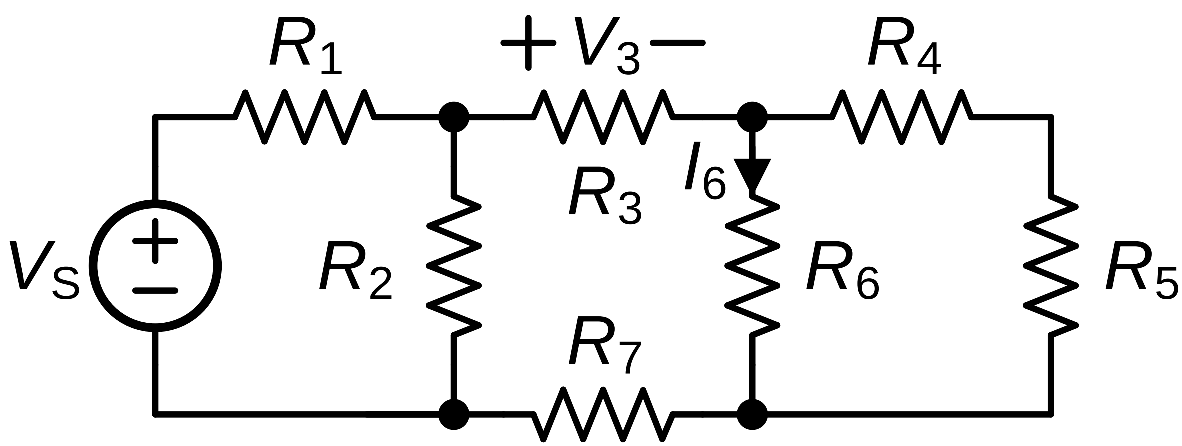
- 并联

$$G_{eq} = G_1 + G_2 \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



例题1

如图所示，已知$R_1 = R_4 = 3\ \Omega$，$R_2 = R_6 = 4\ \Omega$，$R_3 = R_5 = R_7 = 1\ \Omega$，$V_S = 10\ \mathrm{V}$，求电压$V_3$和电流$I_6$。



$$V_3 = 10\ \mathrm{V} \times \frac{4\ \Omega // (1\ \Omega + 4\ \Omega // (3\ \Omega + 1\ \Omega) + 1\ \Omega)}{3\ \Omega + 4\ \Omega // (1\ \Omega + 4\ \Omega // (3\ \Omega + 1\ \Omega) + 1\ \Omega)} \times \frac{1\ \Omega}{1\ \Omega + 4\ \Omega // (3\ \Omega + 1\ \Omega) + 1\ \Omega} = 1\ \mathrm{V}$$

$$I_6 = \frac{1\ \mathrm{V}}{1\ \Omega} \times \frac{3\ \Omega + 1\ \Omega}{4\ \Omega + 3\ \Omega + 1\ \Omega} = 0.5\ \mathrm{A}$$

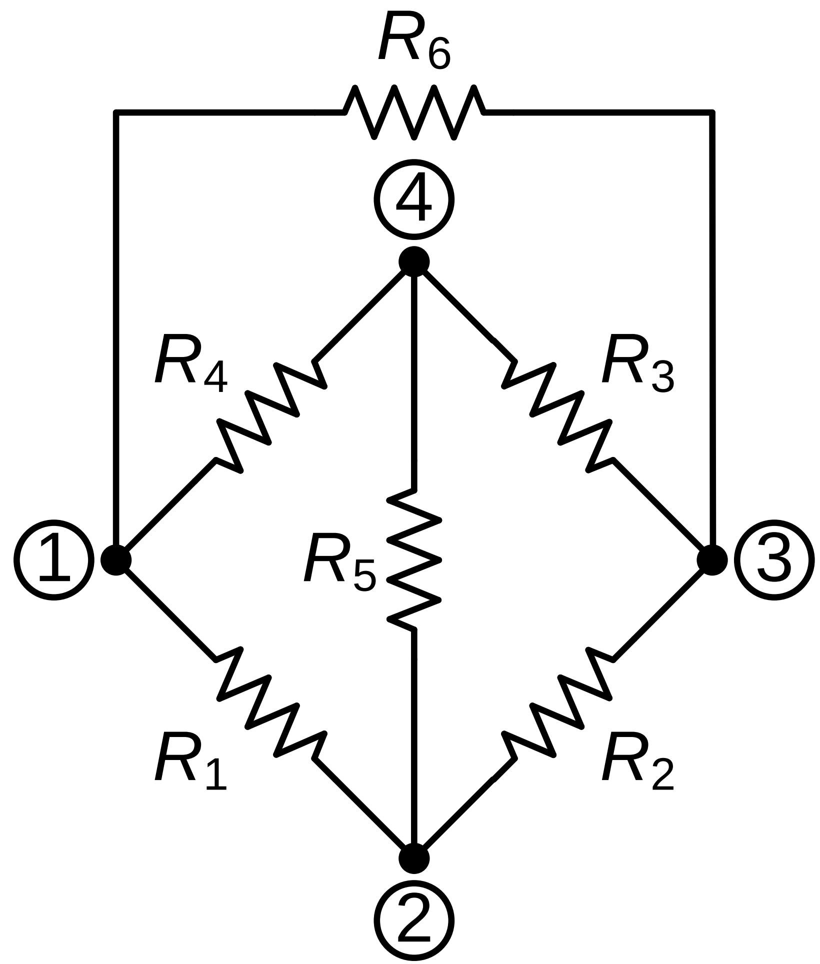
惠斯通电桥

任何两个节点之间由一个电阻与惠斯通电桥电阻并联构成。

例如：节点1和节点3

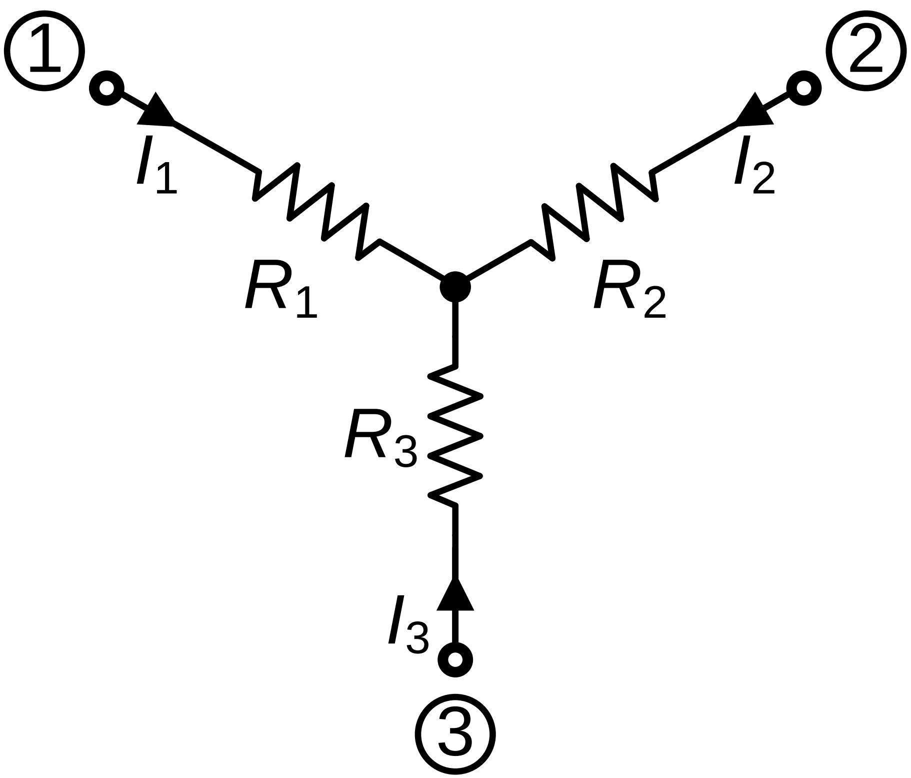
电阻R_1~R_5构成惠斯通电阻，R_6为并联电阻。

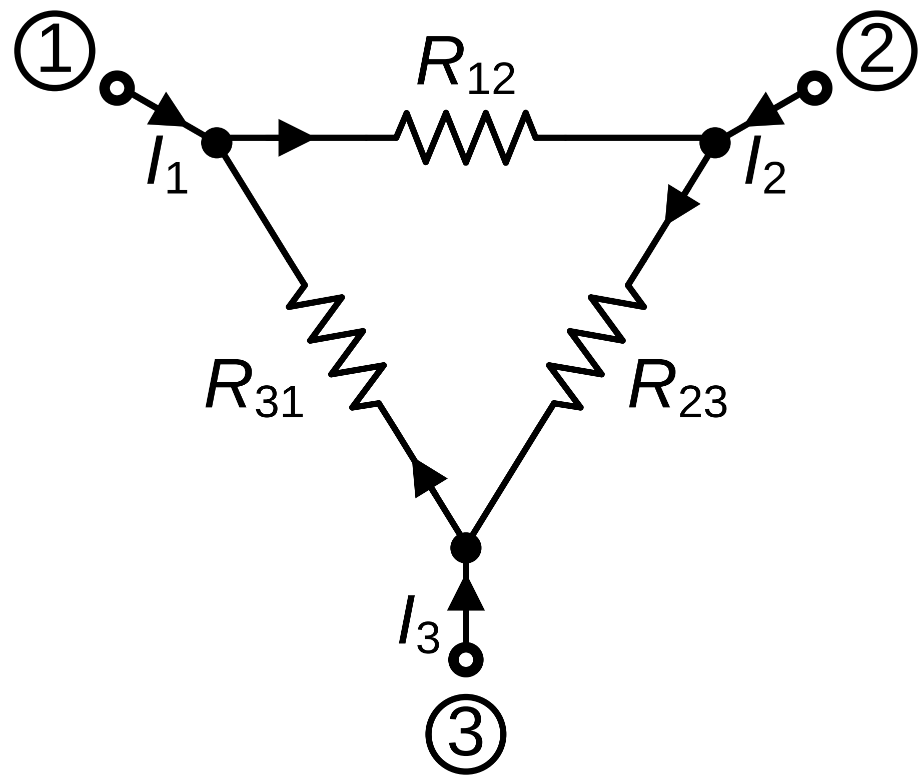
该结构无法通过电阻的串联和并联等效进行简化！



星形和三角形联结的等效

Y形联结和Δ形联结属于三端子网络，不能用串并联等效。





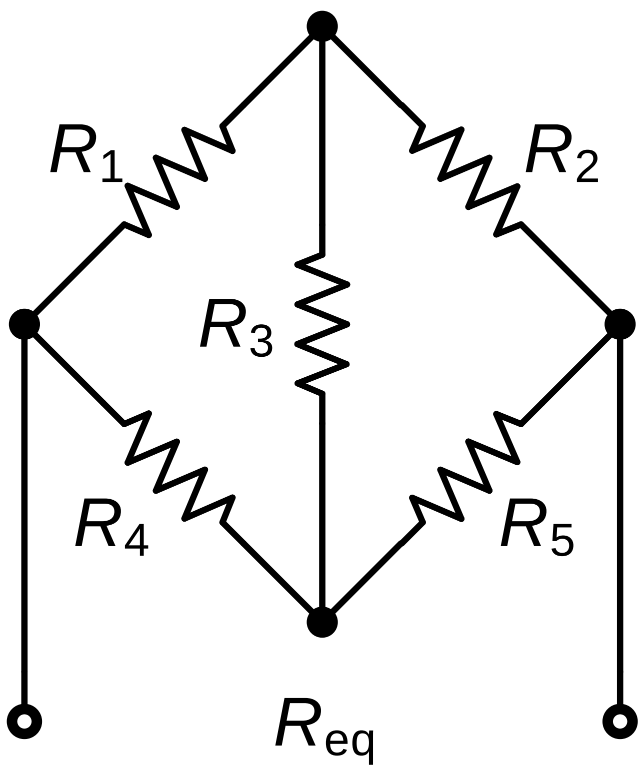
Y形联结	Δ形联结
$\begin{cases} V_{13} = R_1 I_1 - R_3 I_3 = R_1 I_1 + R_3 (I_1 + I_2) \\ V_{23} = R_2 I_2 - R_3 I_3 = R_2 I_2 + R_3 (I_1 + I_2) \end{cases}$	$\begin{cases} I_1 = \dfrac{V_{13}}{R_{31}} + \dfrac{V_{12}}{R_{12}} = \dfrac{V_{13}}{R_{31}} + \dfrac{V_{13} - V_{23}}{R_{12}} \\ I_2 = \dfrac{V_{23}}{R_{23}} - \dfrac{V_{12}}{R_{12}} = \dfrac{V_{23}}{R_{23}} - \dfrac{V_{13} - V_{23}}{R_{12}} \end{cases}$
$\begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_{12}} + \dfrac{1}{R_{31}} & -\dfrac{1}{R_{12}} \\ -\dfrac{1}{R_{12}} & \dfrac{1}{R_{12}} + \dfrac{1}{R_{23}} \end{bmatrix} \begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix}$

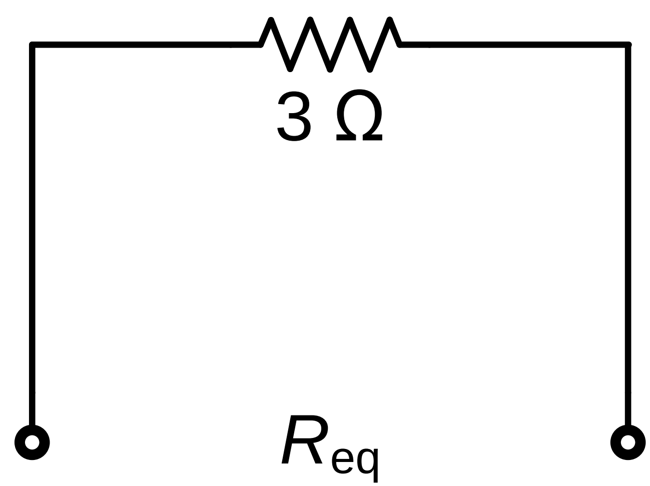
$$\begin{cases} R_{12} = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \\ R_{23} = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_{31} = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{cases} \qquad \begin{cases} R_1 = \dfrac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \\ R_2 = \dfrac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \\ R_3 = \dfrac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \end{cases}$$

若$R_1 = R_2 = R_3 = R_Y$，则$R_\Delta = R_{12} = R_{23} = R_{31} = 3R_Y$，或$R_Y = R_\Delta/3$。

例题2

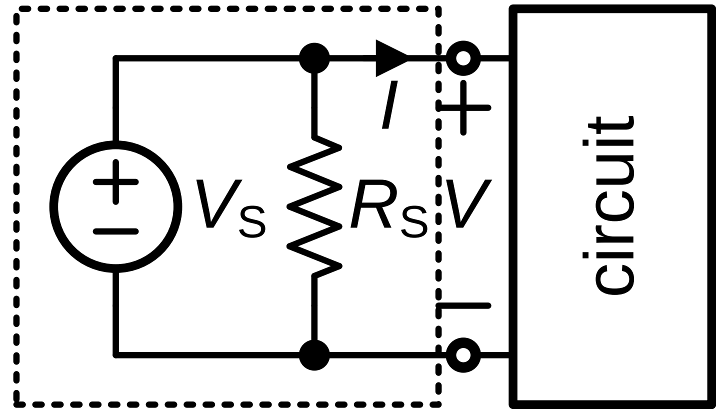
如图所示，已知$R_1 = 3\ \Omega$，$R_2 = 12\ \Omega$，$R_3 = R_4 = R_5 = 2\ \Omega$，求等效电阻$R_{eq}$。



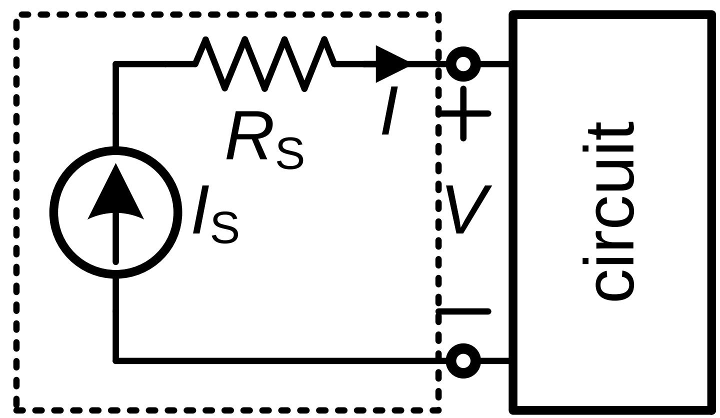


电源和电阻的串联与并联

- 电压源与电阻的并联



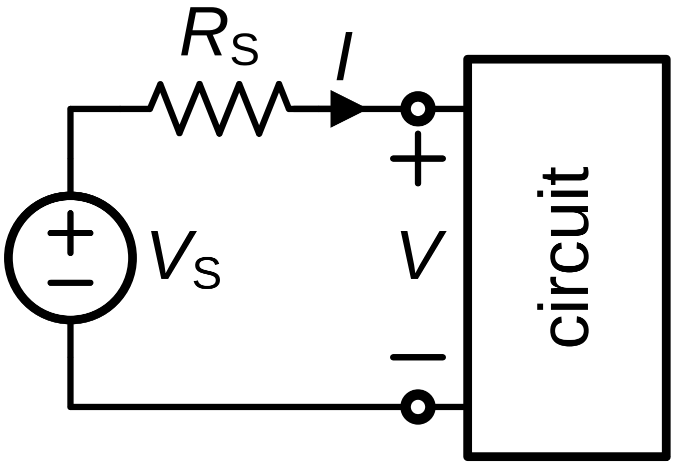
- 电流源与电阻的串联

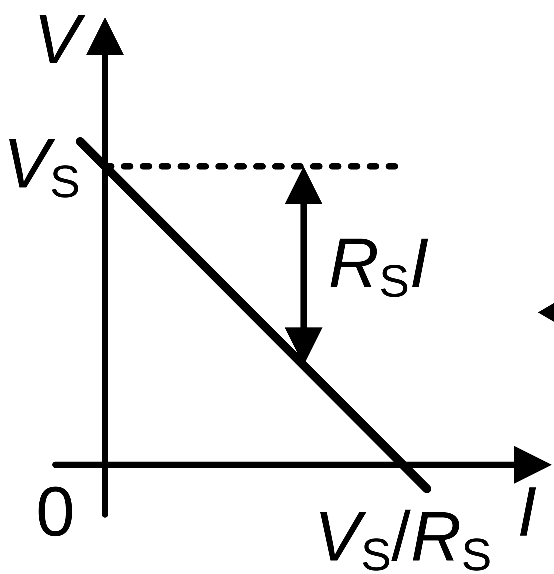


戴维南电路和诺顿电路

戴维南电路

电压源与电阻的串联



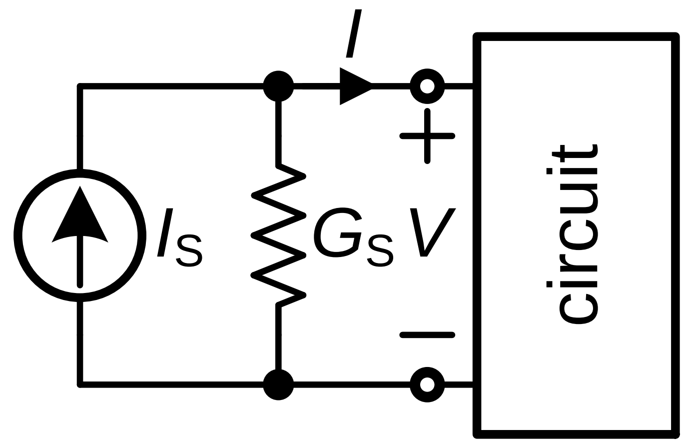


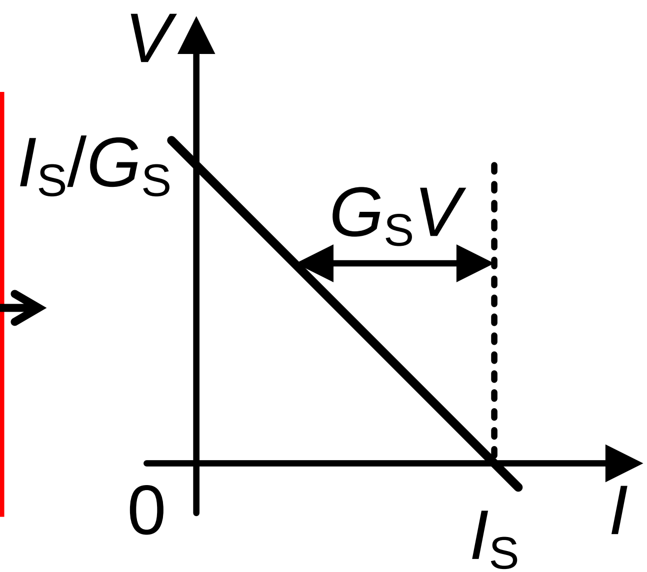
$$I_S = \frac{V_S}{R_S}$$

$$G_S = \frac{1}{R_S}$$

诺顿电路

电流源与电阻的并联

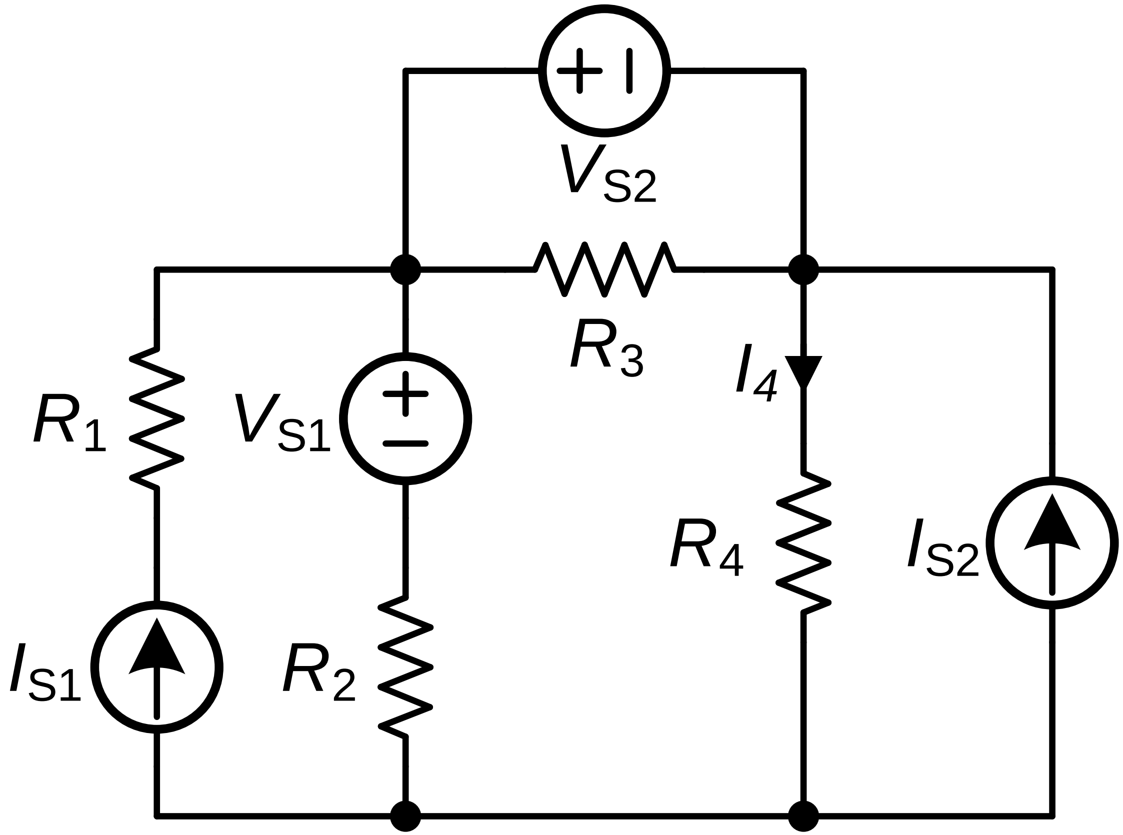


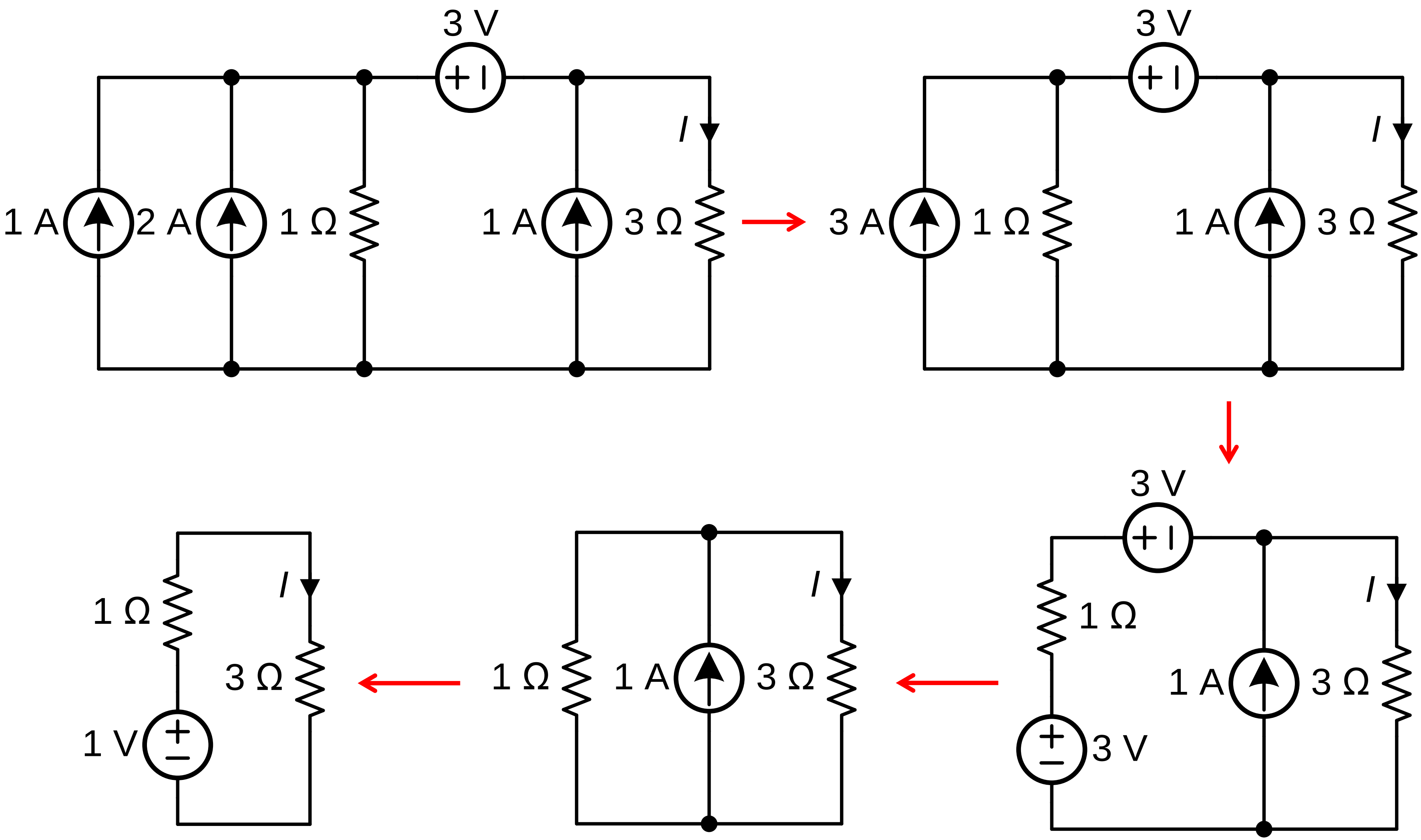


例题3

如图所示，已知$R_1 = 2\ \Omega$，$R_2 = 1\ \Omega$，$R_3 = R_4 = 3\ \Omega$，$V_{S1} = 2\ V$，$V_{S2} = 3\ V$，$I_{S1} = I_{S2} = 1A$，求电流I_4。

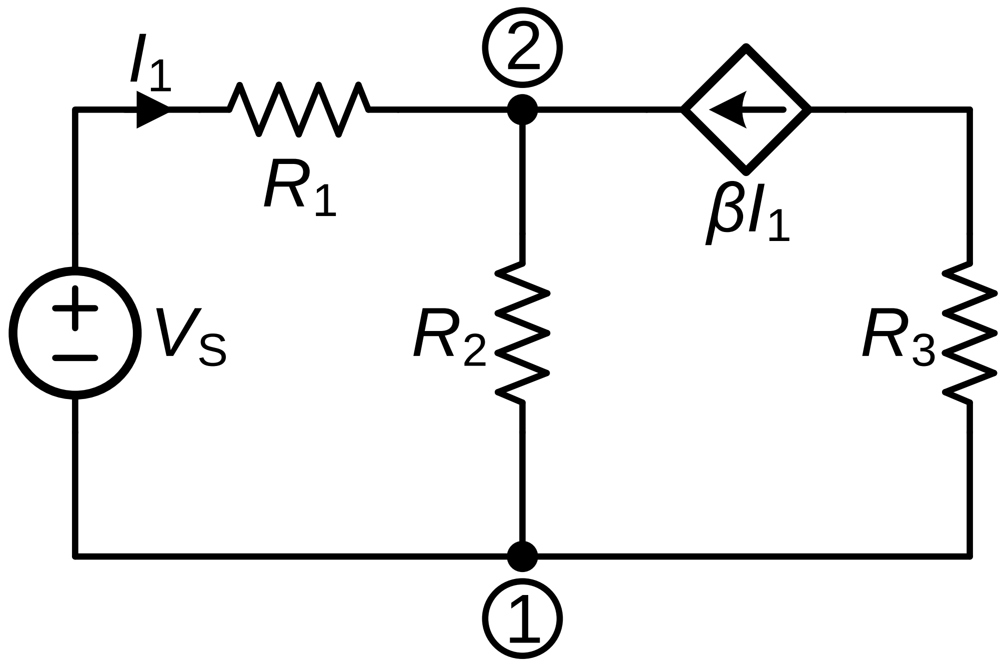
$I_4 = 1/4\ A$




3 V
I
1 A
2 A
1 Ω
1 A
3 Ω
3 V
I
3 A
1 Ω
1 A
3 Ω
3 V
I
1 Ω
1 A
3 Ω
3 V
I
1 Ω
1 A
3 Ω
I
1 Ω
3 Ω
1 V

例题4

如图所示，已知R_1 = 1 Ω，R_2 = 2 Ω，R_3 = 4 Ω，β = 3，V_S = 9 V，求电流I_1。



$$-V_S + I_1(R_1 + R_2) + \beta I_1 R_2 = 0 \qquad I_1 = 1\ \text{A}$$

思考

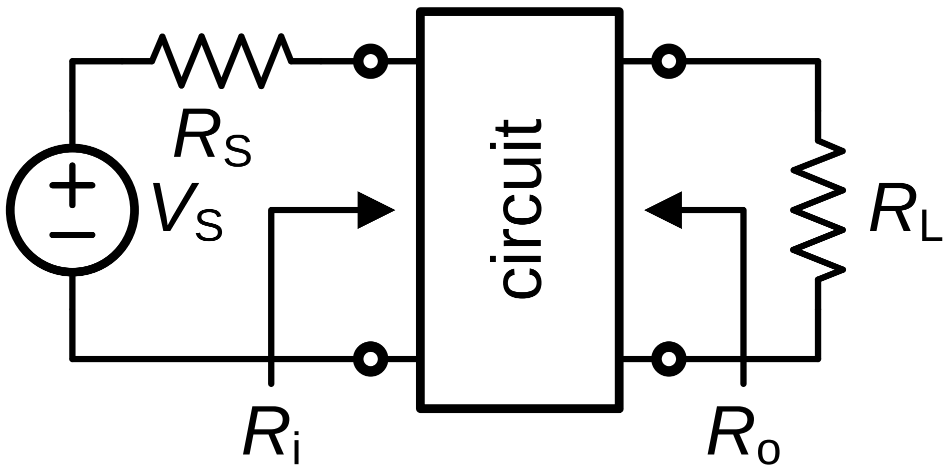
- 电压源的串联和并联

- 电流源的串联和并联

- 电压源和电流源的串联和并联

端口等效电阻

- 输入电阻R_i
- 输出电阻R_o

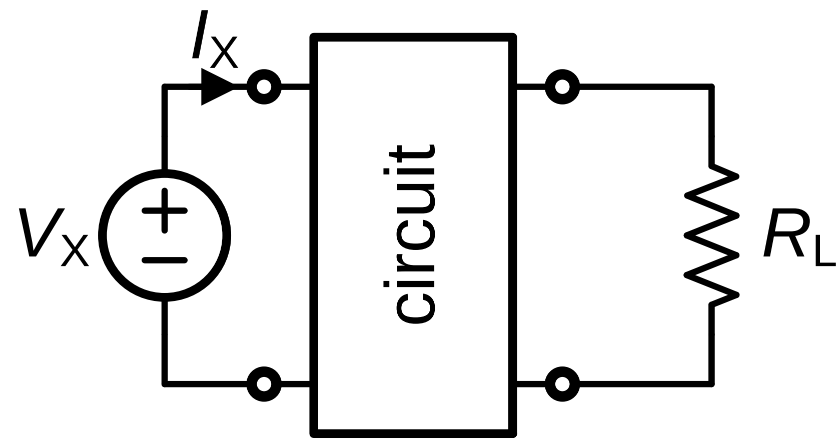


计算端口等效电阻时，独立源置零，受控源保留。独立源置零，就是电压源替换为短路，电流源替换为开路，计算输入电阻时保留负载电阻R_L，计算输出电阻时保留源电阻R_S。

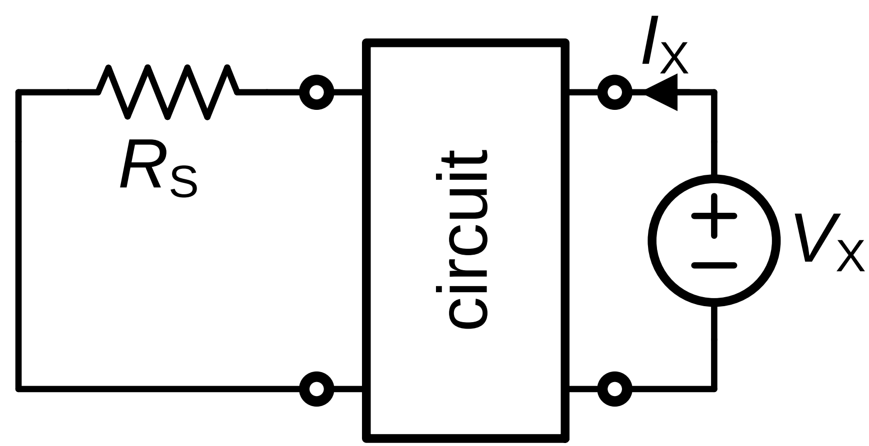
电压源激励法

激励源为电压源V_X，响应量为电流I_X,等效电阻$R_X = V_X/I_X$。

- 输入电阻



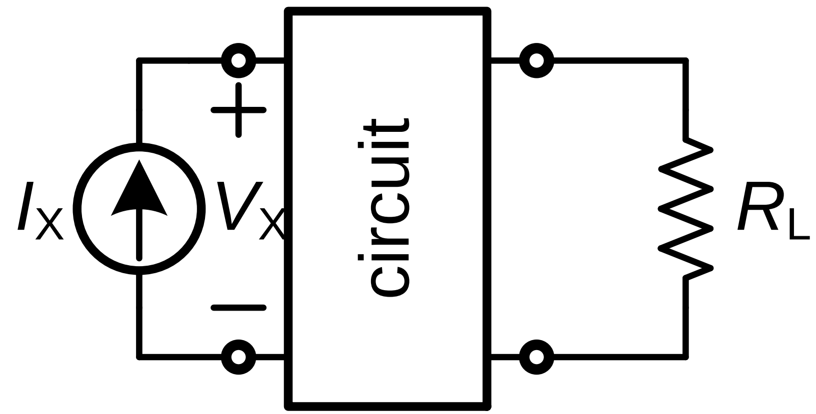
- 输出电阻



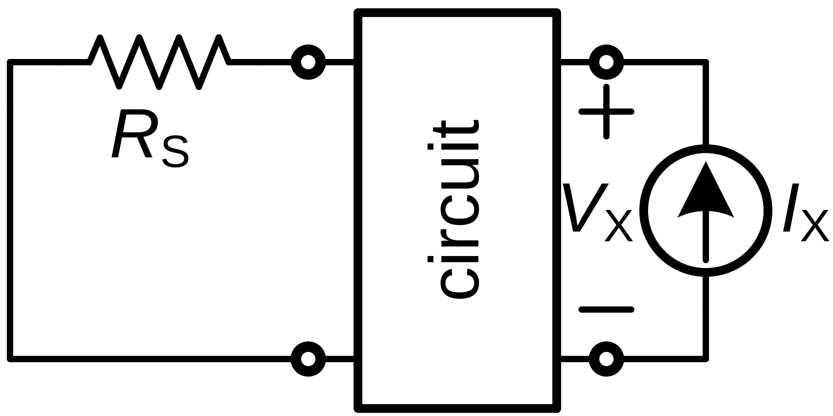
电流源激励法

激励源为电流源I_X，响应量为电压V_X,等效电阻$R_X = V_X/I_X$。

- 输入电阻

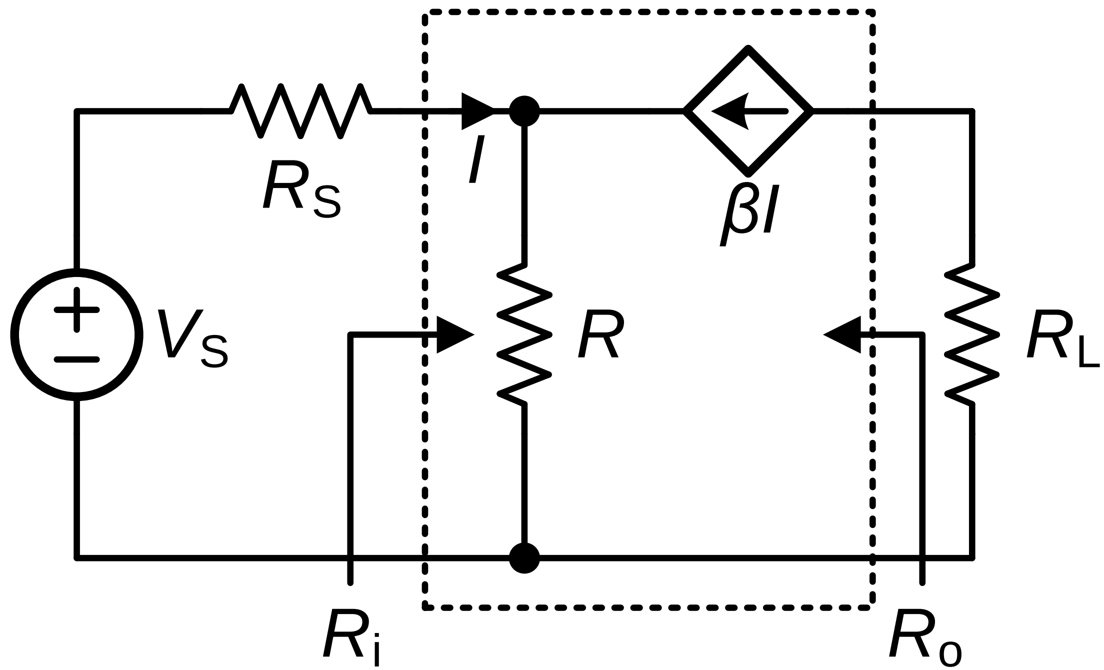


- 输出电阻



例题5

如图所示，已知$R_S = 1\ \Omega$，$R = 2\ \Omega$，$R_L = 4\ \Omega$，$\beta = 3$，求输入电阻R_i和输出电阻R_o。

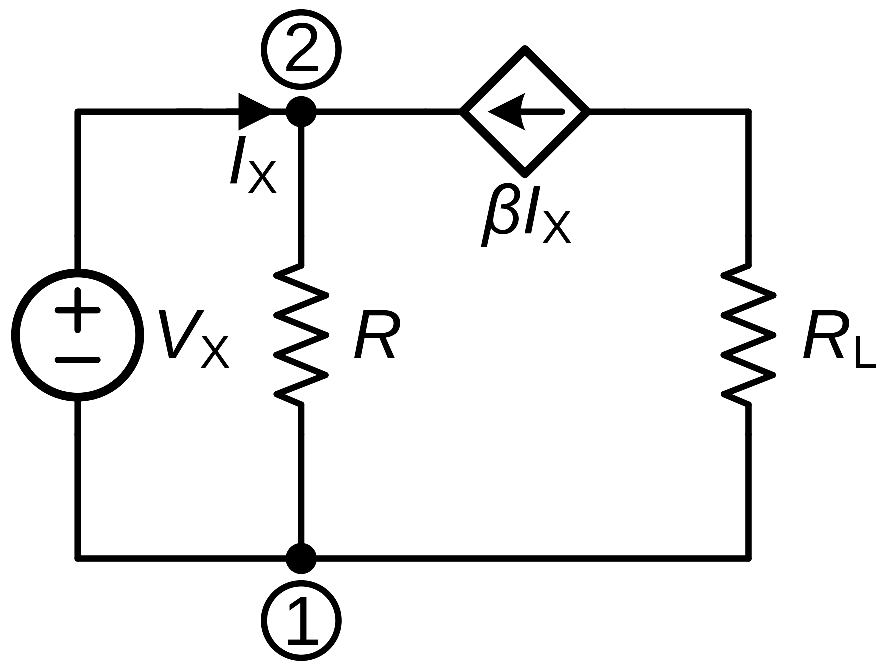


输入电阻R_i

节点2的KCL方程

$$-I_X - \beta I_X + \frac{V_X}{R} = 0$$

$$R_i = \frac{V_X}{I_X} = (1+\beta)R = 8\ \Omega$$



输出电阻R_o

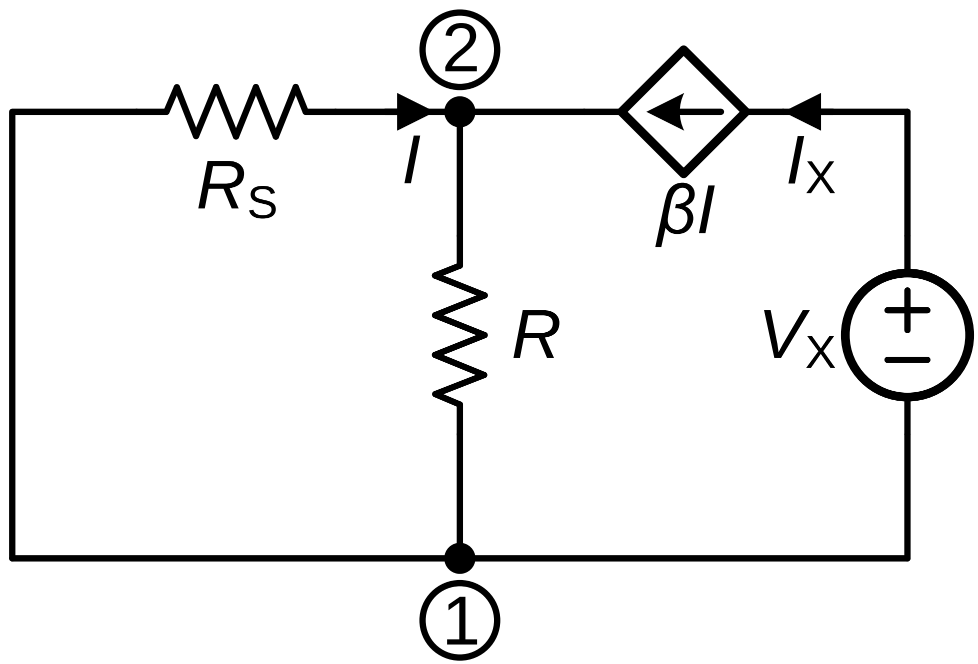
节点2的KCL方程

$$-I-\beta I+\frac{-R_S I}{R}=0$$

$$I=0$$

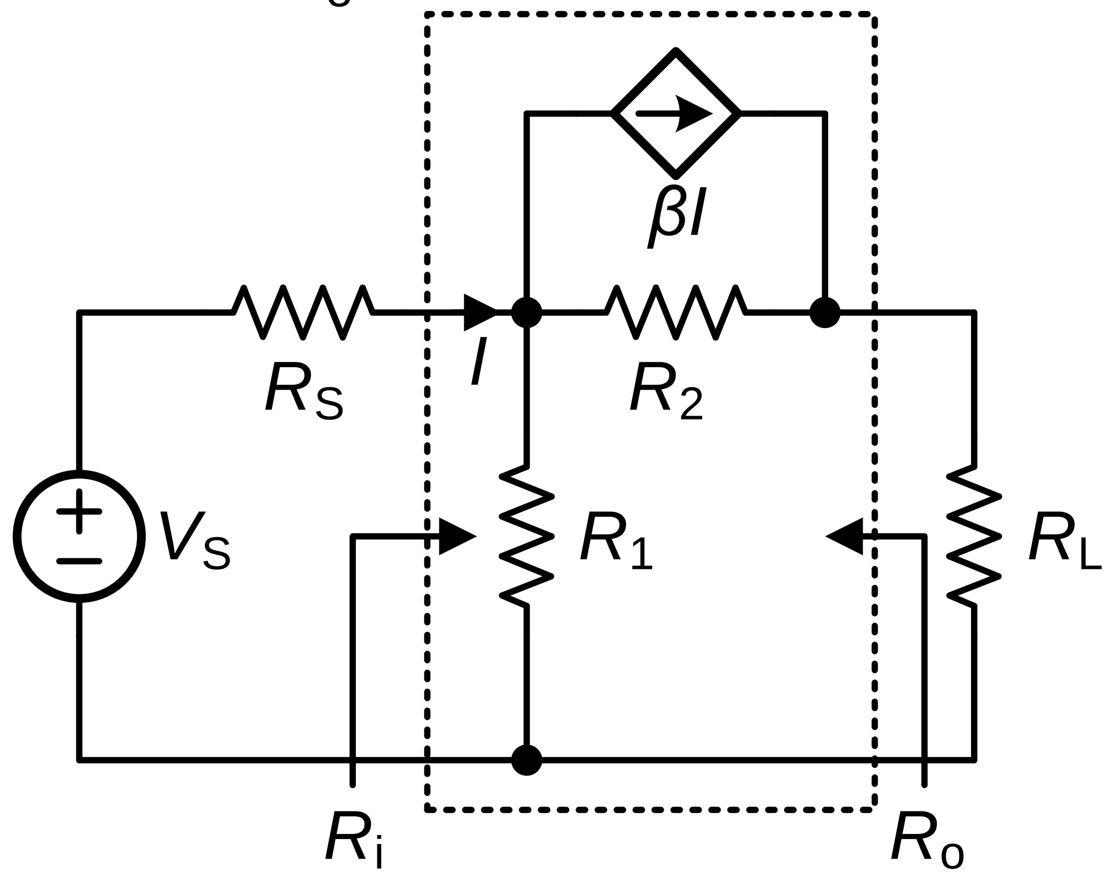
$$I_X=\beta I=0$$

$$R_o=\frac{V_X}{I_X}=\infty$$



例题6

如图所示，已知$R_S = R_1 = R_2 = 1\ \Omega$，$R_L = 2\ \Omega$，$\beta = 5$，求输入电阻$R_i$和输出电阻$R_o$。



输入电阻R_i

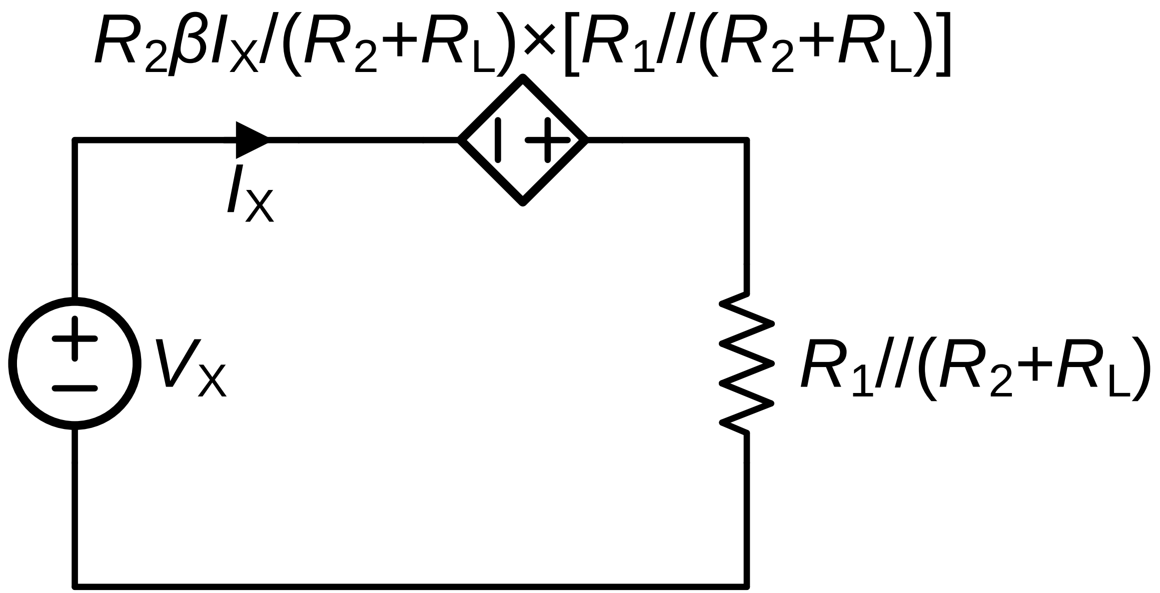
KVL方程

$$-V_X - \frac{R_2\beta I_X}{R_2 + R_L}[R_1 \parallel (R_2 + R_L)] + I_X[R_1 \parallel (R_2 + R_L)] = 0$$

$$R_i = \frac{V_X}{I_X}$$

$$= \frac{R_1[(1-\beta)R_2 + R_L]}{R_1 + R_2 + R_L}$$

$$= -0.5\ \Omega$$



输出电阻R_o

KVL方程

$$-V_X-\frac{\beta I_X R_2 R_1}{R_1+R_S}+I_X[R_2+R_1 \| R_S]=0$$

$$R_o=\frac{V_X}{I_X}$$

$$=\frac{R_S(R_1+R_2)+(1-\beta)R_1R_2}{R_1+R_S}$$

$$=-1\,\Omega$$